Invariance and Matching Market Outcomes

Marek Pycia (UCLA and U Zurich)

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Applicants are asked to submit their ordinal rankings of schools, and school seats are allocated based on these rankings.

The Contentious Choice of Mechanism

- Changes of mechanisms in Boston, NYC, Chicago, New Orleans, Raleigh, and many other districts.
- England's 2007 Admissions Code outlawed the use of non-strategy-proof mechanisms at more than 150 local authorities.
- Recent street protests in France and Taiwan.

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New mechanisms proposed by e.g.: Papai (2000), Bogomolnaia and Moulin (2001), Cres and Moulin (2001), Abdulkadriglu and Sonmez (2001, 2003), Kesten (2010), Pycia and Unver (2017, 2011), Abdulkadiroglu, Che, and Yasuda (forthcoming), Featherstone (2011), Budish, Che, Kojima, Milgrom (2013), Morrill (2014), Ashlagi and Shi (2014), Hakimov and Kesten (2015), He, Miralles, Pycia, and Yan (2015), Nguyen, Peivandi, and Vohra (2015), Shi (2015), and Fisher (2016).

An Empirical Puzzle

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Abdulkadiroglu, Che, Pathak, and Roth (2017):

Table 1. Comparison of Mechanisms in New Orleans for Main Transition Grades (PK and Grade 9)

	TTC-Counters (1)	TTC-Clinch and Trade (2)	Equitable TTC (3)	Serial Dictatorship (4)
		A. Choice Assigned		
1	772	770	771	777
2	126	129	127	121
3	46	47	47	44
4	18	18	18	17
5+	11	11	11	8
Unassigned	222	221	222	228
Total	1196	1196	1196	1196

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			B. Statistics on Blocking Pairs	
Students with justified envy	158	157	159	213
Schools involved in blocking pairs	7	7	7	12
Blocking pairs (i,s)	228	224	215	308
Instances of justified envy (i, (j,s))	1111	1086	1100	6546

An Empirical Puzzle (Boston)

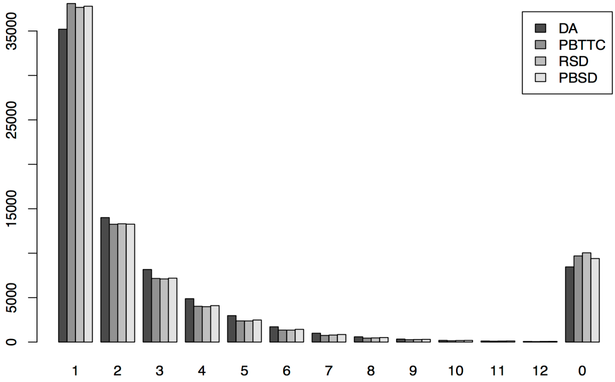
Abdulkadiroglu, Che, Pathak, and Roth (2017):

Table 2. Comparison of Mechanisms in Boston for Main Transition Grades (K1, K2, 6, and 9)

	TTC-Counters (1)	TTC-Clinch and Trade (2)	Equitable TTC (3)	Serial Dictatorship (4)
		A. Choice Assigned		
1	1240	1240	1240	1236
2	322	323	323	315
3	134	134	134	132
4	56	55	55	51
5+	39	39	53	34
Unassigned	102	101	101	124
Total	1893	1893	1893	1893

An Empirical Puzzle (New York)

Che and Tercieux (2018) (average of 100 draws from NYC data):



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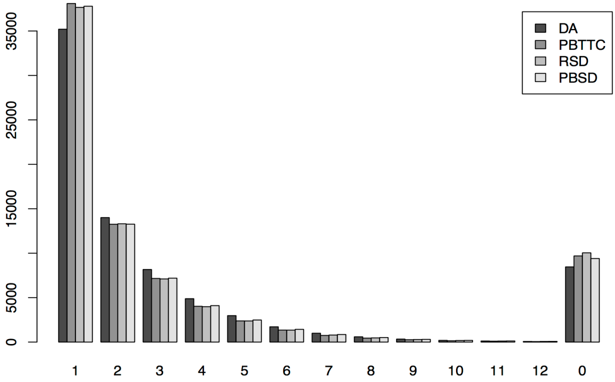

Different allocation scenarios also give similar outcomes—and not only average outcomes—in New York (Abdulkadiroglu, Pathak, and Roth 2009, Abdulkadiroglu, Agarwal, and Pathak 2015), in Boston (Pathak and Sonmez 2008), in Amsterdam (de Haan, Gautier, Oosterbeek, and van der Klaauv 2015), and in New Orleans (Pathak 2016).

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- Many standard mechanisms—e.g. Serial Dictatorships and Top Trading Cycles—generate exactly the same mean invariant statistics if agents' preferences are drawn from exchangeable distributions.

- Analogous results hold true for stable mechanisms.

- Methodological innovations (Duality of Symmetry, bringing tools from econometrics).

Literature

Outcome equivalence for symmetric and strategy-proof mechanisms: Abdulkadiroglu and Sonmez (1998), Kesten (2006), Che and Kojima (2010), Miralles (2008), Pathak and Sethuraman (2011), Liu and Pycia (2011), Lee and Sethuraman (2011), Ashlagi and Shi (2014), Carroll (2014), Ekici (2014), Pycia and Troyan (2016).

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Che and Tercieux (2018): asymptotic equivalence of theoretical payoff distributions for strategy-proof mechanisms.

Model

A – finite set of schools; each school $a \in A$ has $|a| > 0$ seats.

N – finite set of agents; each agent i demands a single seat and has a strict preference ranking $\succ_i$ over schools.

Θ is the set of agents' unobservable types, e.g. preference rankings.

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A *mechanism* ϕ maps profiles of messages to allocations. For notational simplicity, suppose agents report strict rankings of schools (this is not crucial).

A profile of strategies $\hat{\succ}_N$ in mechanism ϕ is in *Nash equilibrium* if, for any agent i, reporting $\hat{\succ}_i$ weakly dominates reporting any $\succ_i'$ (pure strategies).

Outcome Statistics

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Statistics F is *invariant* if $F\left((\theta_i, a_i)_{i \in I}\right)$ depends only on the empirical distribution of $f\left(\theta_i, \phi\left(\succ\left(\theta_i\right), i\right)\right)_{i \in I}$ and the dependence is linear.

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Examples of invariant statistics:

- how many students obtain their top outcome, their two top outcomes, etc;
- the empirical distribution of ranks;
- the empirical distribution of payoffs;
- how many students are assigned to school A, school B, etc.

Robustness

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Deterministic mechanisms such as serial dictatorships, top trading cycles, Papai's (2001) hierarchical exchange and Pycia and Unver's (2017, 2001) trading cycles with fixed endowments are robust with $c = |A|$.

Main Theorem (1)

Theorem. Let statistics F be invariant. For every $\epsilon > 0$ and for sufficiently large $|N|$ (relative to ϵ and $|A|$), for any two Pareto efficient Nash equilibria $\succ^{\phi}$, $\succ^{\psi}$ of robust mechanisms ϕ and ψ and for at least $1 - \epsilon$ fraction of all preference profiles

$$\sum_{\ell=1}^{|K|} \left| F_\ell \left(\succ, \phi \left(\succ^{\phi} \right) \right) - F_\ell \left(\succ, \psi \left(\succ^{\psi} \right) \right) \right| < \epsilon.$$

Main Theorem (2)

Fix any $\delta > 0$ and define

$$\mathcal{P}_{\delta,N} = \{\succ_N\colon\ (\forall \succ)\ |\{i \in N :\succ_i = \succ\}| > \delta\}\,.$$

Theorem. For any $|K|\,, |A| \in \mathbb{N}$ and $c, \delta > 0$, there is $C > 0$ such that the invariant outcome statistics of any two Pareto efficient Nash equilibria $\succ^{\phi}$ and $\succ^{\psi}$ of any ratio-c robust mechanisms ϕ_N, ψ_N are approximately equal:

$$|F_1(\succ_N, \phi_N) - F_1(\succ_N, \psi_N)| < \frac{C}{|N|},$$

except for at most fraction $2e^{-\left(\frac{C}{12c}\right)^2|N|}$ of preference profiles.

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When $|K| = 2$, we can take $C = 8\left(\frac{2+|A|}{\delta}\right)^{\frac{5}{2}} \sqrt{c\,|N|}$. This is a rough bound; in particular, we can use C that doesn't depend on $|N|$.

Sketch of the Proof

Stochastic mechanism ϕ gives probabilities $\phi(i, \succ)(a)$ that i obtains a. The *symmetrization* ϕ^S of mechanism ϕ gives

$$\phi^S(i, \succ)(a) = \sum_{\sigma : N \overset{1-1}{\to} N} \frac{1}{|N|!} \phi(\sigma(i), \succ_\sigma)(a).$$

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1. For all $\epsilon > 0$, sufficiently large N, at least $1 - \epsilon$ fraction of preference profiles, and any strategy-proof, Pareto-efficient, and continuous mechanisms ϕ^S and ψ^S:

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(Liu and Pycia 2011; Nash relies on the additional structure here).

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2. For exchangeable distributions of preferences, the mean of an invariant statistics F of ϕ and ψ are asymptotically the same (Duality of Symmetry).

3. The realizations of invariant statistics are asymptotically equivalent for asymptotically almost all preference profiles (tools from econometrics).

Exchangeable Distributions

A distribution of types is *exchangeable* if the probability of the type profile θ_N is the same as the probability of the profile $\theta_{\sigma(N)}$ for any permutation $\sigma : N \to N$.

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E.g. the uniform preference distributions studied by Lee (2014), Ashlagi, Kanoria, and Leshno (2017), Lee and Yariv (2015), Che and Tercieux (2018) are exchangeable.

Duality of Symmetry

Theorem (Asymptotic Duality of Symmetry). Take two sequences of direct mechanisms, ϕ_N and ψ_N, such that their symmetrizations have asymptotically equivalent marginal distributions of outcomes. If agents' types are drawn from an exchangeable distribution then the mean of any invariant statistics of outcomes under ϕ_N is asymptotically the same as under ψ_N.

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Approximate versions (and proper inverse) of this duality are also true.

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The mean of $F\left(\left(\succ_{\sigma(i)}, \phi\left(\sigma\left(i\right), \succ_{\sigma(I)}\right)\right)_{i\in N}\right)$ over a uniform distribution over σ is equal to the mean of $F\left(\left(\succ_i, \phi^S\left(i, \succ_I\right)\right)_{i\in N}\right)$; and by the law of iterated expectations the same obtains for any exchangeable distribution over σ; and hence for any exchangeable distribution over types $\succ_N$.

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The same holds for ψ and ψ^S. Because $\phi^S = \psi^S$, the claim obtains.

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- Follows from the Duality of Symmetry + Abdulkadiroglu and Sonmez (1998) (for house allocation), and Pathak and Sethuraman (2011) (for school choice).

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All Pareto efficient and strongly obviously strategy-proof mechanisms have the same mean invariant outcome statistics.

- Duality of Symmetry + Pycia and Troyan (2016).

Random Mechanisms

The results extend to random mechanisms provided the equilibria are in dominant strategies.

Priorities: Model

Let K be the set of priority types. The priority types $K(i)$ and $K(j)$ of two agents i and j determines their priority ranking at each school: at each school they might be of the same type or one might have higher priority, and the ranking may be different at different schools.

An allocation is *stable* if there is no pair of agents i and j such that i has higher priority at the school j is assigned and i prefers this school over his or her assignment.

Priorities: Result

Theorem. Let F be an invariant outcome statistics. For every $\epsilon > 0$ and for sufficiently many agents in each priority group (relative to ϵ and $|A|$), for any two stable and constrained-Pareto-efficient Nash equilibria $\succ^{\phi}$, $\succ^{\psi}$ of robust mechanisms ϕ and ψ and for at least $1-\epsilon$ fraction of all preference profiles

$$\sum_{\ell=1}^{|K|} \left| F_{\ell}\left(\succ, \phi\left(\succ^{\phi}\right)\right) - F_{\ell}\left(\succ, \psi\left(\succ^{\psi}\right)\right) \right| < \epsilon.$$

(It is enough to assume robustness locally at $\succ^{\phi}$, $\succ^{\psi}$).

Conclusions (1)

Are mechanisms studied equivalent from welfare perspective?

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No, they differ with respect to non-invariant outcome statistics such as:

- Improvements over reference mechanism (He 2011, Calsamiglia and Miralles 2012, Agarwal and Somaini 2016).
- Violations of stability (Kesten 2010, Abdulkadiroglu, Che, Pathak, and Roth 2017).
- Lorenz dominance (Pycia and Unver 2014, Harless and Manjunath 2016).
- Arrovian welfare comparisons (Pycia and Unver 2016).

Conclusions (2)

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• To construct better mechanisms we need to elicit preference intensity as in: Hylland and Zeckhauser (1979), Abdulkadiroglu, Che, and Yasuda (forthcoming), He, Miralles, Pycia, and Yan (2015), Azevedo and Budish (2015), Ashlagi and Shi (2015), Miralles and Pycia (2014, 2015), and Nguyen, Peivandi, and Vohra (2016).

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- Mechanisms eliciting preference-intensity do better than purely ordinal ones: Miralles (2008), Abdulkadiroglu, Che, and Yasuda (2011), Featherstone and Niederle (2016), Troyan (2014), Pycia (2014), Ashlagi and Shi (2015), Abdulkadiroglu, Agarwal, and Pathak (2016), Eraslan, Fox, He, and Pycia (2017).

Summary

- An explanation of the empirical puzzle and new testable implications.
- The invariant outcome statistics of any robust, strategy-proof, and Pareto efficient mechanisms are asymptotically the same, profile-by-profile.
- Many standard mechanisms are exactly equivalent in terms of mean invariant outcome statistics.