Invariance and Matching Market Outcomes

Marek Pycia (UCLA and U Zurich)
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Allocation of school seats to students

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Applicants are asked to submit their ordinal rankings of schools, and school seats are allocated based on these rankings.
The Contentious Choice of Mechanism

- Changes of mechanisms in Boston, NYC, Chicago, New Orleans, Raleigh, and many other districts.
- England’s 2007 Admissions Code outlawed the use of non-strategy-proof mechanisms at more than 150 local authorities.
- Recent street protests in France and Taiwan.
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An Empirical Puzzle
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Abdulkadiroglu, Che, Pathak, and Roth (2017):

Table 1. Comparison of Mechanisms in New Orleans for Main Transition Grades (PK and Grade 9)

<table>
<thead>
<tr>
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Che and Tercieux (2018) (average of 100 draws from NYC data):

Different allocation scenarios also give similar outcomes—and not only average outcomes—in New York (Abdulkadiroglu, Pathak, and Roth 2009, Abdulkadiroglu, Agarwal, and Pathak 2015), in Boston (Pathak and Sonmez 2008), in Amsterdam (de Haan, Gautier, Oosterbeek, and van der Klaauw 2015), and in New Orleans (Pathak 2016).
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- Analogous results hold true for stable mechanisms.
- Methodological innovations (Duality of Symmetry, bringing tools from econometrics).

Model

\(A\) – finite set of schools; each school \(a \in A\) has \(|a| > 0\) seats.

\(N\) – finite set of agents; each agent \(i\) demands a single seat and has a strict preference ranking \(\succ_i\) over schools.

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A *mechanism* $\phi$ maps profiles of messages to allocations. For notational simplicity, suppose agents report strict rankings of schools (this is not crucial).

A profile of strategies $\hat{\succ}_N$ in mechanism $\phi$ is in *Nash equilibrium* if, for any agent $i$, reporting $\hat{\succ}_i$ weakly dominates reporting any $\succ'_i$ (pure strategies).
Outcome Statistics

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\[ f : \Theta \times A \to K = \{1, \ldots, k\} \text{ for some } k \in \mathbb{N}. \]
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Statistics \( F \) is *invariant* if \( F\left((\theta_i, a_i)_{i \in I}\right) \) depends only on the empirical distribution of \( f\left(\theta_i, \phi(\succ (\theta_i), i)\right)_{i \in I} \) and the dependence is linear.
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Examples of invariant statistics:

- how many students obtain their top outcome, their two top outcomes, etc;
- the empirical distribution of ranks;
- the empirical distribution of payoffs;
- how many students are assigned to school A, school B, etc.
Robustness

A deterministic mechanism is robust with ratio $c > 0$ if changing the report of one agent affects at most $c$ other agents.
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A deterministic mechanism is *robust* with ratio $c > 0$ if changing the report of one agent affects at most $c$ other agents.

Deterministic mechanisms such as serial dictatorships, top trading cycles, Papai’s (2001) hierarchical exchange and Pycia and Unver’s (2017, 2001) trading cycles with fixed endowments are robust with $c = |A|$. 
Theorem. Let statistics $F$ be invariant. For every $\epsilon > 0$ and for sufficiently large $|N|$ (relative to $\epsilon$ and $|A|$), for any two Pareto efficient Nash equilibria $\succ^\phi$, $\succ^\psi$ of robust mechanisms $\phi$ and $\psi$ and for at least $1 - \epsilon$ fraction of all preference profiles

$$\sum_{\ell=1}^{\|K\|} \left| F_{\ell} \left( \succ, \phi \left( \succ^\phi \right) \right) - F_{\ell} \left( \succ, \psi \left( \succ^\psi \right) \right) \right| < \epsilon.$$
Main Theorem (2)

Fix any $\delta > 0$ and define

$$\mathcal{P}_{\delta,N} = \{\succ_N: (\forall \succ) \left| \{i \in N : \succ_i = \succ \} \right| > \delta \}.$$ 

**Theorem.** For any $|K|, |A| \in \mathbb{N}$ and $c, \delta > 0$, there is $C > 0$ such that the invariant outcome statistics of any two Pareto efficient Nash equilibria $\succ^\phi$ and $\succ^\psi$ of any ratio-$c$ robust mechanisms $\phi_N, \psi_N$ are approximately equal:

$$|F_1(\succ_N, \phi_N) - F_1(\succ_N, \psi_N)| < \frac{C}{|N|},$$

except for at most fraction $2e^{-\left(\frac{C}{12c}\right)^2|N|}$ of preference profiles.
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When $|K| = 2$, we can take $C = 8 \left(\frac{2 + |A|}{\delta}\right)^{\frac{5}{2}} \sqrt{c |N|}$. This is a rough bound; in particular, we can use $C$ that doesn’t depend on $|N|$.
Sketch of the Proof

Stochastic mechanism $\phi$ gives probabilities $\phi(i, \succ) (a)$ that $i$ obtains $a$. The symmetrization $\phi^S$ of mechanism $\phi$ gives

$$\phi^S (i, \succ) (a) = \sum_{\sigma : N \rightarrow N} \frac{1}{|N|!} \phi (\sigma (i), \succ \sigma) (a).$$
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(Liu and Pycia 2011; Nash relies on the additional structure here).
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2. For exchangeable distributions of preferences, the mean of an invariant statistics $F$ of $\phi$ and $\psi$ are asymptotically the same (Duality of Symmetry).

3. The realizations of invariant statistics are asymptotically equivalent for asymptotically almost all preference profiles (tools from econometrics).
Exchangeable Distributions

A distribution of types is *exchangeable* if the probability of the type profile $\theta_N$ is the same as the probability of the profile $\theta_{\sigma(N)}$ for any permutation $\sigma : N \rightarrow N$. 

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Duality of Symmetry

Theorem (Asymptotic Duality of Symmetry). Take two sequences of direct mechanisms, \( \phi_N \) and \( \psi_N \), such that their symmetrizations have asymptotically equivalent marginal distributions of outcomes. If agents’ types are drawn from an exchangeable distribution then the mean of any invariant statistics of outcomes under \( \phi_N \) is asymptotically the same as under \( \psi_N \).
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Proof of Duality of Symmetry

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For any agent $i$, the probability $f (\succ_{\sigma(i)}, \phi (\sigma (i), \succ_{\sigma(I)})) = 1$ with $\sigma$ distributed uniformly over all permutations is equal to the probability $f (\succ_i, \phi^S (i, \succ_I)) = 1$ where $\phi^S$ is the symmetrization of over $\phi$. 
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Because $\phi^S = \psi^S$, the claim obtains.
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The same holds for $\psi$ and $\psi^S$. Because $\phi^S = \psi^S$, the claim obtains.
Applications

Serial Dictatorship and Top Trading Cycles have exactly the same mean invariant outcome statistics.
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• Follows from the Duality of Symmetry + Abdulkadiroglu and Sonmez (1998) (for house allocation), and Pathak and Sethuraman (2011) (for school choice).
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- Duality of Symmetry + Pycia and Troyan (2016).
Random Mechanisms

The results extend to random mechanisms provided the equilibria are in dominant strategies.
Let $K$ be the set of priority types. The priority types $K(i)$ and $K(j)$ of two agents $i$ and $j$ determines their priority ranking at each school: at each school they might be of the same type or one might have higher priority, and the ranking may be different at different schools.

An allocation is *stable* if there is no pair of agents $i$ and $j$ such that $i$ has higher priority at the school $j$ is assigned and $i$ prefers this school over his or her assignment.
Priorities: Result

**Theorem.** Let $F$ be an invariant outcome statistics. For every $\epsilon > 0$ and for sufficiently many agents in each priority group (relative to $\epsilon$ and $|A|$), for any two stable and constrained-Pareto-efficient Nash equilibria $\succ^\phi, \succ^\psi$ of robust mechanisms $\phi$ and $\psi$ and for at least $1 - \epsilon$ fraction of all preference profiles

$$\sum_{\ell=1}^{\lfloor K \rfloor} \left| F_\ell \left( \succ, \phi \left( \succ^\phi \right) \right) - F_\ell \left( \succ, \psi \left( \succ^\psi \right) \right) \right| < \epsilon.$$  

(It is enough to assume robustness locally at $\succ^\phi, \succ^\psi$.)
Conclusions (1)

Are mechanisms studied equivalent from welfare perspective?

• Improvements over reference mechanism (He 2011, Calsamiglia and Miralles 2012, Agarwal and Somaini 2016).
• Violations of stability (Kesten 2010, Abdulkadiroglu, Che, Pathak, and Roth 2017).
• Lorenz dominance (Pycia and Unver 2014, Harless and Manjunath 2016).
• Arrovian welfare comparisons (Pycia and Unver 2016).
Conclusions (1)

Are mechanisms studied equivalent from welfare perspective? No, they differ with respect to non-invariant outcome statistics such as:

- Arrovian welfare comparisons (Pycia and Unver 2016).
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- In large markets we cannot improve upon the simple Serial Dictatorship if we only elicit ordinal rankings.
- To construct better mechanisms we need to elicit preference intensity as in: Hylland and Zeckhauser (1979), Abdulkadiroglu, Che, and Yasuda (forthcoming), He, Miralles, Pycia, and Yan (2015), Azevedo and Budish (2015), Ashlagi and Shi (2015), Miralles and Pycia (2014, 2015), and Nguyen, Peivandi, and Vohra (2016).
Conclusions (2)

If we primarily care about invariant outcome statistics:

• In large markets we cannot improve upon the simple Serial Dictatorship if we only elicit ordinal rankings.

• To construct better mechanisms we need to elicit preference intensity as in: Hylland and Zeckhauser (1979), Abdulkadiroglu, Che, and Yasuda (forthcoming), He, Miralles, Pycia, and Yan (2015), Azevedo and Budish (2015), Ashlagi and Shi (2015), Miralles and Pycia (2014, 2015), and Nguyen, Peivandi, and Vohra (2016).

Summary

• An explanation of the empirical puzzle and new testable implications.

• The invariant outcome statistics of any robust, strategy-proof, and Pareto efficient mechanisms are asymptotically the same, profile-by-profile.

• Many standard mechanisms are exactly equivalent in terms of mean invariant outcome statistics.