Directed Search and the Futility of Cheap Talk

Kenneth Mirkin and Marek Pycia

June 2015. Preliminary Draft.

Abstract

We study directed search in a frictional two-sided matching market in which each seller can communicate with buyers prior to matching via costless public messages that entail no obligation regarding post-matching behavior (cheap talk). We establish that messages cannot contain payoff-relevant information in any equilibrium.

1. Introduction

We explore the capacity of cheap-talk communication to direct search in a frictional two-sided matching market. Each seller is endowed with an indivisble good that each buyer values, and the two sides are matched via some process from a general class of (potentially multilateral) matching technologies. Both sides are heterogeneous and cannot observe others' types prior to matching; only within each match do seller types become observable, while buyer types remain private information throughout. Once matched, the seller can choose and implement any mechanism to allocate her good. While each seller can communicate with buyers prior to matching via costless public messages, these messages entail no obligation regarding post-matching behavior.

In this fairly general setting, we establish that messages cannot contain payoff-relevant information in any equilibrium. This is in stark contrast to a variety of existing work in similar environments demonstrating the feasibility of informative sorting equilibria without pre-match commitment. Our result holds broadly across matching technologies and persists even in the presence of type complementarities; as such, this highlights limitations to the scope of positive cheap-talk results in similar contexts and further clarifies the fundamental role of public commitment in facilitating efficient sorting in market settings.

2. Context in Literature

- Kim and Kircher "Efficient Competition through Cheap Talk: The Case of Competing Auctions" (ECTA 2015)
- Doyle and Wong "Wage Posting without Full Commitment" (RED 2013)
- Coles and Eeckhout "Indeterminacy and Directed Search" (JET 2003)
- Eeckhout and Kircher "Sorting versus Screening: Search Frictions and Competing Mechanisms" (JET 2010)
- Menzio "A Theory of Partially Directed Search" (JPE 2007)
- Michelacci and Suarez "Incomplete Wage Posting" (JPE 2006)
- Geromichalos "Directed Search and the Bertrand Paradox" (IER 2014)

3. Model

Setting and Notation

Sellers can post any message $m \in \mathcal{M}$ (where $\mathcal{M}$ is a finite space) and, in turn, buyers can apply to any message in this space. The sellers who have posted and the buyers who have applied to a particular message m are matched according to a potentially multilateral technology, resulting in a stochastic number $n \in 0 \cup \mathbb{N}$ of buyers arriving at each seller (add citation?). For simplicity, we consider the probabilistic distribution behind each seller's number of buyers to depend only on the market tightness (buyer-seller ratio $\theta(m)$) for the corresponding message; we denote this distribution for each k by $\Pr(n = k \,|\theta(m))$.

Buyer types: $x \in [\underline{x}, \overline{x}] \equiv X$

Seller types: $y \in [\underline{y}, \overline{y}] \equiv Y$

Highest type buyer and seller at message m: $\overline{x}(m)$, $\overline{y}(m)$

Lowest type buyer and seller at message m: $\underline{x}(m)$, $\underline{y}(m)$

Note: *We assume both $f^b(\widetilde{x})$ and $f^s(\widetilde{y})$ are Lebesque absolutely continuous*

(exogenous) distribution of buyer types: $f^b(\widetilde{x})$

(exogenous) distribution of seller types: $f^s(\widetilde{y})$

(endogenous) distribution of buyer types at each message m: $g^b(\widetilde{x}, m)$

(endogenous) distribution of seller types at each message m: $g^s(\widetilde{y}, m)$

The total masses of buyers and sellers are $B = \int_{\underline{x}}^{\overline{x}} f^b(\widetilde{x})\, d\widetilde{x} = \widehat{\theta}$ and $S = \int_{\underline{y}}^{\overline{y}} f^s(\widetilde{y})\, d\widetilde{y}$, respectively. Let us normalize the seller mass to be $S = 1$ and parametrize the buyer mass to be $B = \widehat{\theta}$, so $\widehat{\theta}$ denotes the aggregate buyer-to-seller population ratio.

We restrict attention (WLOG) to second price auctions with a reserve price. Matching technology is continuous.

Assume: $\overline{y} < \overline{x}$: so that every seller sells has a positive probability of benefitting from selling to a higher value buyer. This precludes our equilibrium from being impacted by the reserve prices of sellers who never benefit from trade in equilibrium.

4. Results

Let us introduce some notation to aid in this section's arguments. Let $V(y, m)$ denote the expected payoff of a type y seller, conditional on posting the message m, and let $V(y)$ denote this type's unconditional expected payoff. In turn, let $U(x, m)$ denote the expected payoff of a type x buyer, conditional on visiting a seller who has posted the message m, and let $U(x)$ denote the corresponding unconditional expected payoff. Additionally, define $\pi(x, m)$ to be the probability that a type x buyer attending message m wins or ties with another buyer. More precisely, $\pi(x, m) \equiv \lim_{\widetilde{x} \longrightarrow x^+} \Pr(\text{ win } |\widetilde{x}, m)$.

Lastly, we want to introduce the notion of a highest type buyer receiving no expected surplus at each message. Formally, for each message $m \in \mathcal{M}$, define the set $X_0(m) \subset X$ by $X_0(m) = \{\widetilde{x} \in X \,|U(\widetilde{x}, m) = 0\}$, and define $x_L(m) = \sup_{\widetilde{x} \in X_0(m)} \{\widetilde{x}\}$.

Before presenting the main result, we first provide a few intermediate findings which we will call upon in the proof. Let us briefly introduce some notation to aid in the arguments which follow.

Lemma 1: *$U(x,m)$ is strictly monotonically increasing and continuous in x for each $m \in \mathcal{M}$*

Proof: The strict monotonicity is obvious—a type $x+\varepsilon$ buyer could always mimic the behavior of a type x buyer, and even if the type $x+\varepsilon$ buyer's win probability is no greater than that of the type x buyer, type $x+\varepsilon$ will enjoy a strictly higher payoff each time she wins the object.

This holds regardless of the possibility of mass points in buyer type distributions or reserve price distributions at each message.

The probability of winning might jump discretely when buyer type distribution mass points are present, but at a type just above the discontinuity, winning buyers obtain virtually zero surplus whenever such wins are a specific consequences of the discontinuity. This is because, in such cases, the winning buyer must defeat another with a type arbitrarily close to (and below) her own.

Similarly, the probability of winning might jump discretely due to a mass point of reserve prices, but winning buyers obtain approximately zero surplus from the incremental wins which specifically reflect the discontinuity—such wins involve paying a reserve price which is virtually identical to the buyer value. ■

Lemma 2: *$\forall m \in \mathcal{M}$, $x_L(m) = x_L$*

Proof: Suppose not, so that there are m_1, $m_2 \in \mathcal{M}$ and corresponding $x_L(m_1)$, $x_L(m_2) \in X$ where $x_L(m_1) < x_L(m_2)$ Naturally, type $x_L(m_1)$ obtains zero surplus at m_1 and $x_L(m_2)$ obtains zero surplus at m_2. Because of this, though—along with the strict monotonicity and continuity of $U(x,m)$ in x—strictly positive surplus must be available to types $x \in [x_L(m_2) - \varepsilon, x_L(m_2) + \varepsilon]$ (with small $\varepsilon > 0$) at m_1, while such types could achieve only zero surplus (for types $x \in [x_L(m_2) - \varepsilon, 0])$ or infinitessimal surplus (for types $x \in (0, x_L(m_2) + \varepsilon])$ at m_2 due to Lemma 1. Hence, these types would search only at message m_1. Not facing any buyers in this range, sellers at message m_2 would have no reason to post reserve prices below $x_L(m_2) + \varepsilon$. Of course, this would contradict our originally defined $x_L(m_2)$, as the true supremum of types receiving zero surplus at m_2 would be higher than this.

Thus, for any pair of messages m_1, $m_2 \in \mathcal{M}$, it must be that $x_L(m_1) = x_L(m_2)$. From this, it is immediate that all x_L must be the same at all $m \in \mathcal{M}$. ■

Theorem (Main Result): *In any Perfect Bayesian Equilibrium, $U(x,m) = U(x)$ for all buyer types $x \in X$ and each $m \in \mathcal{M}$.*

Proof: Consider a buyer of some type $x_1 \in X$ who strictly prefers some message m_1 to another message m_2 (i.e. - $U(x,m_1) > U(x,m_2)$). From the continuity part of Lemma 1, there must be some interval of types around x_1—say, $(x_1 - \varepsilon, x_1 + \varepsilon)$—within which buyers strictly prefer message m_1 to m_2. We know that type $x_L < x_1$ is indifferent between m_1 and m_2 so we know that $U(x,m_1)$ and $U(x,m_2)$ must meet at at least one point in $[x_L, x_1]$. Let x_0 denote the closest type below x_1 for whom this holds.

Clearly, buyer types $x \in (x_0, x_1]$ strictly prefer m_1 to m_2.

Claim: $\pi(x_0, m_1) \geq \pi(x_0, m_2)$

Toward contradiction, suppose $\pi(x_0, m_1) < \pi(x_0, m_2)$. Focusing still on type x_0, let us consider the incremental increase in expected payoffs at each of these two messages, as buyer type increases locally from x_0 to $x_0 + dx$. Obviously

$dx < x_1 - x_0$ so no buyers with types in the range $[x_0, x_0 + dx]$ visit message m_2 (because m_1 strictly dominates it). Then we can represent the aforementioned incremental payoff changes for each message with the expressions:

$$U(x_0 + dx, m_1) - U(x_0, m_1) = \pi(x_0, m_1)\, dx + \delta(x_0, x_0 + dx, m_1)$$
$$U(x_0 + dx, m_2) - U(x_0, m_2) = \pi(x_0, m_2)\, dx + \underbrace{\delta(x_0, x_0 + dx, m_2)}_{0}$$

where $\delta(x, x + dx, m)$ correspond to the expected utility gained at message m when the buyer wins against types or reserve prices in the interval $(x, x + dx)$. Concerning $\delta(x_0, x_0 + dx, m_2)$, note that there are no additional buyers to beat in this range, as m_1 is strictly preferred. Without buyers in this range, sellers have no reason to post reserve prices anywhere but at $x_0 + dx$ itself. Even such reserve prices, however, offer a type $x_0 + dx$ buyer no opportunity for a positive payoff. Hence, $\delta(x_0, x_0 + dx, m_2) = 0$.

Additionally, the expressions above combined with the strict preference for m_1 imply the following relationship:

$$[\pi(x_0, m_2) - \pi(x_0, m_1)]\, dx < \delta(x_0, x_0 + dx, m_1)$$

Notice, however, that $\delta(x_0, x_0 + dx, m_1)$ is second order in dx. Indeed, $\cap_{dx>0}(x_0, x_0 + dx) = \emptyset$ implies that the probability of winning against a buyer or reserve price in $(x_0, x_0 + dx)$ goes to zero as dx goes to 0; and the gain conditional on winning against these types is at most first order in dx. For small dx, then, this contradicts the above inequality. ✓

Claim: Given x_0 such that (a) $U(x_0, m_1) = U(x_0, m_2)$, (b) $\pi(x_0, m_1) \geq \pi(x_0, m_2)$, and (c) some $x_1 > x_0$ such that $x \in (x_0, x_1]$ strictly prefer m_1 to m_2, no buyers with types $x > x_0$ visit m_2 or m_1 in equilibrium.

As above, let us consider the incremental payoff changes at each message, working upward from x_1.

$$U(x_1 + dx, m_1) - U(x_1, m_1) = \pi(x_1, m_1)\, dx + \delta(x_1, x_1 + dx, m_1)$$

$$U(x_1 + dx, m_2) - U(x_1, m_2) = \pi(x_1, m_2)\, dx$$

Note that the strict preference for m_1 in this range enables us to extend the inequality established in the previous claim to type x_1: $\pi(x_1, m_1) \geq \pi(x_1, m_2)$. This, combined with the fact that $\delta(x, x + dx, m_1)$ is nonnegative, precludes the incremental payoff increase at m_2 from ever exceeding that at m_1. Because $U(x_1, m_1) > U(x_1, m_2)$, we can push this locally from x_1 all the way to $\overline{x}$ to establish that $U(\overline{x}, m_1) > U(\overline{x}, m_2)$ as well. Hence, it must be that no buyers above x_0 visit m_2.

Given that no buyers with types $x > x_0$ visit m_2 and our assumption that $\overline{y} < \overline{x}$, no sellers will set reserve prices at m_2 above x_0 because these offer an expected payoff of zero, and all sellers have an opportunity to obtain positive payoffs in equilibrium. Hence $\pi(x_0, m_2) = 1$. But since $\pi(x_0, m_1) \geq \pi(x_0, m_2)$, this means $\pi(x_0, m_1) = 1$ also! ✓

We have thus established that, given two messages and a type x buyer who strictly prefers one message to the other, there must be no buyers at or above this type visiting either of these two messages in equilibrium. *(We've actually established something slightly stronger that extends up from the nearest type below who is indifferent between the messages).*

Claim: Consider $x \in [\underline{x}, \overline{x}]$ and $m \in \arg\max_{\widetilde{m}} \{U(x, \widetilde{m})\}$. $\nexists m' \in \mathcal{M}$ for which $U(x, m') < U(x, m)$.

For $x \in [\underline{x}, x_L]$, we've already established this.

Now consider $x \in (x_L, \overline{x}]$. Suppose this doesn't hold, so that such an m' does exist. Notice that, using m' pairwise with each message among those which are most-preferred, the above argument can be extended to establish that no types at or above x can visit any message in $\arg\max_{\widetilde{m}} \{U(x, \widetilde{m})\}$. If this holds for all messages among those which are optimal for a type x buyer, there is nowhere for type x buyers to go, so we have a contradiction. ✓

Certainly, if no message can be strictly worse for each buyer type than any of the optimal messages, then each type must be indifferent across all messages. ■

References

[1] Doyle, Matthew and Wong, Jacob. 2013. "Wage Posting without Full Commitment." *Review of Economic Dynamics*, 16(2): 231-252.

[2] Coles, Melvyn G. and Eeckhout, Jan 2003. "Indeterminacy and Directed Search." *Journal of Economic Theory*, 111(2): 265-276.

[3] Eeckhout, Jan and Kircher, Philipp. 2010. "Sorting and Decentralized Price Competition." *Econometrica*, 78(2): 539-574.

[4] Eeckhout, Jan and Kircher, Philipp. 2010. "Sorting versus Screening: Search Frictions and Competing Mechanisms." *Journal of Economic Theory*, 145(4): 1354–1385.

[5] Galenianos, Manolis and Kircher, Philipp. 2012. "On the Game-Theoretic Foundations of Competitive Search Equilibrium." *International Economic Review*, 53(1): 1-21.

[6] Geromichalos, Athanasios. 2014. "Directed Search and the Bertrand Paradox." *International Economic Review*, 55(4): 1043-1065.

[7] Kim, Kyungmin and Kircher, Philipp. 2015. "Efficient Competition through Cheap Talk: The Case of Competing Auctions." *Econometrica*, forthcoming.

[8] Benjamin Lester, Ludo Visschers, and Ronald Wolthoff. 2015 "Meeting Technologies and Optimal Trading Mechanisms in Competitive Search Markets." *Journal of Economic Theory*, 155: 1-15.

[9] McAfee, Preston. 2002. "Coarse Matching." *Econometrica*, 70(5): 2025-2034.

[10] Menzio, Guido. 2007. "A Theory of Partially Directed Search." *The Journal of Political Economy*, 115(5): 748-769.

[11] Michelacci, Claudio and Suarez, Javier. 2006. "Incomplete Wage Posting." *The Journal of Political Economy*, 114(6): 1098-1123.

[12] Peters, Michael. 1991. "Ex Ante Price Offers in Matching Games: Non-Steady States." *Econometrica*, 59(5): 1425-1454.

[13] Peters, Michael. 1997. "A Competitive Distribution of Auctions." *The Review of Economic Studies*, 64(1): 97-123.