Outside Options and the Failure of the Coase Conjecture

Simon Board and Marek Pycia*

July 8, 2011

Abstract

A buyer wishes to purchase a good from a seller who chooses a sequence of prices over time. In each period, the buyer can also exercise an outside option such as moving onto another seller. We show there is a unique equilibrium in which the seller charges a constant price in every period equal to the monopoly price against their residual demand. This result contravenes the Coase conjecture.

1 Introduction

The Coase conjecture is a cornerstone of modern microeconomic theory, informing monopoly theory and providing a canonical example of the problem of commitment. The idea is that, for any given price, high value buyers are more likely to purchase than low value buyers, leading to a negative selection in the demand pool. As a result, the seller cuts its price over time, causing high value buyers to delay their purchases. The seller's inability to commit thus leads its later selves to exert a negative externality on its former selves, reducing their overall profit (Gul, Sonnenschein and Wilson (1987)). This idea of negative selection is robust: versions of it hold when costs are nonlinear (Kahn (1986)), when goods depreciate over time (Bond and Samuelson (1984)), when there is entry of new agents (Sobel (1991)) and when the buyers face future competition (Fuchs and Skrzypacz (2010)).

In this paper, we show that the Coase conjecture fails in a natural environment where buyers have outside options. We consider a seller who faces a buyer (or a continuum of buyers) with unknown values for the good and a competing alternative. For example, consider a household that wishes to complete a construction project and receives quotes from a contractor. They have outside options coming from within the market (e.g. other contractors) and outside the market (e.g. the household may finish the project themselves).

In Section 2, we take the outside options as exogenous and show that there is unique equilibrium in which the seller charges a constant price equal to the monopoly price against the residual demand. The fact that such a monopolistic equilibrium exists is not so surprising. If buyers expect the price

*Department of Economics, UCLA. We thank Andy Skrzypacz and Joel Sobel for feedback.

to stay high then there is no point waiting until the next period for a discount. As a result, there is no negative selection in the demand pool and no price cut is forthcoming.

More surprisingly, we show that the monopoly pricing equilibrium is unique. Consider the type of buyer with the lowest net value from consuming the good, $\underline{u}$. In any equilibrium, this buyer cannot obtain positive utility since the seller will never lower its price below $\underline{u}$. Hence, the agent should always take the outside option rather than wait for a second price quote, causing the set of waiting buyers to unravel from the bottom. The overall idea is that the existence of outside options creates an opportunity cost of waiting for low value buyers, and results in positive selection in the demand pool. This is very natural: when shopping in a bazaar, a high price quote from a seller is more likely to lead a low value buyer to move onto the next stall rather than wait for a price reduction.

High-priced equilibria have been produced in other durable goods models. Ausubel and Deneckere (1989) show that if buyers expect an out-of-equilibrium price cut to be followed by Coasian pricing, then one can sustain prices that are arbitrarily high. Fudenberg, Levine and Tirole (1987) suppose the seller has the option to consume the good and show that if the buyer expects the seller to consume early, he is less willing to delay, causing the seller to become more pessimistic over time and justifying the belief that the seller consumes early. In both of these cases there are multiple equilibria, including those with Coasian dynamics; in our paper, the monopoly equilibrium is unique.

In Section 3, we put the result into context by considering a model of sequential search. Each period, the buyer chooses whether stay with the current seller, or move onto an alternative seller where he receives a new value draw; the chosen seller then quotes a price as a function of its history with the buyer. In this case, the value from future search opportunities constitutes the outside alternative and allows us to apply our monopoly pricing result. In equilibrium, a seller faces buyers who arrive over time, receive one offer, buy if their value is sufficiently high and otherwise move on.

This model thus shows that while the Coase conjecture applies with a single seller, it fails to hold with multiple sellers. This result complements Ausubel and Deneckere (1987) and Gul (1987) who consider direct competition between price-setting sellers. These models have multiple equilibria ranging from traditional Bertrand pricing to monopolistic pricing that is sustained through Bertrand punishments. In contrast, our model has a unique equilibrium and does not rely on non-Markovian punishments that might be hard to coordinate. This significantly simplifies the analysis: once we have proved that sellers choose constant prices, the results are similar to Anderson and Renault (1999), albeit with proportional discounting.

2 Main Result

We start by looking at how the presence of the buyer's outside option changes equilibrium play in an otherwise standard Coase conjecture setting. A monopolistic seller tries to sell a durable good. The seller's cost of producing the good is $c = 0$, and is commonly known. There is one buyer who privately knows his value for the good $v \in V \subset [0, \infty)$ and the value of his outside option

$w \in W \subset [\underline{w}, \infty)$, where $\underline{w} > 0$. The values (v, w) are drawn from a distribution with support contained in $V \times W$; the distribution is commonly known.[1]

Time is discrete, $t \in \{1, 2, \ldots\}$. At the start of any period t, the seller chooses price p_t. The buyer then chooses whether to buy the good, exercise his outside option, or wait. The game continues only if the buyer chooses to wait (all actions are publicly observable).

Waiting is costly as agents discount their utility with a common discount factor $\delta \in (0, 1)$. If the seller sells the good in period t, it obtains profit $\delta^t p_t$. If the buyer buys the good in period t, he obtains utility $\delta^t (v - p_t)$; if he exercises the option in period t, he obtains $\delta^t w$. If the buyer always waits, both seller and buyer obtain 0.

Define $u := v - w$ as the *net value* of the buyer. Without loss of generality we may assume that $u \geq 0$ as buyers with negative net value will exercise their outside option in any equilibrium. Let $F(u)$ be the cdf of the buyer's net value.

A *Perfect Bayesian Equilibrium (PBE)* is a history-contingent sequence of the seller's offers $\{p_t\}$, the buyer's acceptance and exercise decisions, and of updated beliefs about the buyer's values (v, w) satisfying the usual consistency conditions: the actions are optimal given the beliefs, and the beliefs are derived from the actions by Bayes' rule.[2] An equilibrium is *essentially unique* if all equilibria lead to the same payoffs. Essential uniqueness allows a buyer who is indifferent to choose as they see fit, and does not pin down prices off the equilibrium path. The buyer and seller follow *monopoly strategies* if in each period

1. The seller charges $p^m \in \text{argmax } p\,(1 - F\,(p))$.

2. A buyer buys the good if $u \geq p$, and otherwise exercises his outside option.

Suppose we reached some history h in the game, and some types of buyers remain. Define $\underline{u} = \underline{u}\,(h)$ as the essential infimum of the support of net values, $v - w$, for (v, w) from the remaining distribution. That is, $\underline{u}$ is the infimum of u such that there is a positive mass of agents with net values below u.

Lemma 1. *In any PBE, given any on-path history h, the price charged by the seller after any on-path continuation of history h is above $\underline{u}\,(h)$.*

Proof. Fix a PBE and an on-path history h_0, and let H be the set of continuations h of history h_0 such that after any sub-history of h at which the buyer takes an action, the buyer's action is from the support of actions prescribed by the PBE. Thus, histories in H may involve seller's deviations but no buyer's deviations. In particular, after any history from H, the seller puts probability 1 on buyer's net value being weakly above $\underline{u} = \underline{u}\,(h)$. Let $\underline{p}$ be the essential infimum of prices the

[1] This setup is quite general: The distribution over (v, w) is not restricted, so may have gaps and atoms. The value v may be commonly known to be above some lowest positive value (the "gap-case" of the Coase conjecture); in fact, the value may be commonly known to be large, say $v > 100$, and the outside option may be commonly known to be small, say $w = 1$. We suppose there is a single buyer, but one can also assume the seller sells a continuum of goods to a continuum of buyers with different values and outside options.

[2] The beliefs are derived by the Bayes' rule whenever possible, including off-equilibrium path; and no seller's action, not even zero-probability action, changes her belief about the buyer's type.

seller charges after histories from H. Since H contains all on-path continuations of h_0, to prove the lemma, it is enough to show that $\underline{p} \geq \underline{u}$.

By way of contradiction, assume that $\underline{p} < \underline{u}$. Fix $\epsilon < (1-\delta)(\underline{u} - \underline{p})$ and consider a history $h \in H$ after which the seller moves and there is positive probability that the seller charges a price from the interval $\left[\underline{p}, \underline{p} + \epsilon\right)$. Let p_h be the random price charged after history h.

Now, consider a deviation in which after history h the seller charges $\tilde{p}_h = \max\{p_h, \underline{p} + \epsilon\}$. Conditional on $p_h \geq \underline{p} + \epsilon$ the deviation has no effect on the subsequent play and on profits. However, if $p_h < \underline{p} + \epsilon$, then the seller believes that the buyer immediately buys under both new and old prices, so the seller's profit is strictly higher under the new prices. To see this, notice that after her deviation the seller still believes that $v - w \geq \underline{u}$, and – as we are about to show – this implies that the buyer immediately buys at any price $p \leq \underline{p} + \epsilon$. The buyer prefers to buy rather than exercise his outside option because $\underline{u} > \underline{p} + \epsilon \geq p$ implies $v - p > w$. He prefers to buy rather than wait because $\underline{u} - p \geq \underline{u} - (\underline{p} + \epsilon) > \delta(\underline{u} - \underline{p})$ implies that $v - p > \delta(v - \underline{p})$, and he expects future prices to be weakly above $\underline{p}$ (because the equilibrium histories of the game following the deviation belong to H). □

Proposition 1. *There is an essentially unique PBE in which the buyer and seller use monopoly strategies.*

Proof. To check that the constructed profile of strategies is a PBE is straightforward: If the buyer exits or buys in period $t = 1$, the seller will charge the monopoly price; If the seller charges the monopoly price forever, then the seller will exit or buy in period $t = 1$.

To demonstrate the payoff-equivalence of all PBE, it is enough to show that in $t = 1$ the seller charges the monopoly price, and buyers buy or exercise their outside option, whichever is more profitable (indifferent buyers can do either). To demonstrate this by way of contradiction, assume that there is a PBE in which some positive-measure subset of buyer's types decides to wait until period $t = 2$. Let $\underline{u}$ be the essential infimum of the net values of buyers that decide to wait.

By definition, there is a positive mass of buyers with net value below $\underline{u} + \epsilon$ that decided to wait. By waiting, these low value buyers have foregone or postponed taking their w. Lemma 1 tells us that the seller will always charge a price of at least $\underline{u}$. Since taking the outside option next period cannot be better than taking it now, these low value buyers would only wait if they prefer to buy the good at price $\underline{u}$ next period over taking w now, that is if

$$w \leq \delta\left(v - \underline{u}\right) \leq \delta\left(\left(w + \underline{u} + \epsilon\right) - \underline{u}\right) = \delta\left(w + \epsilon\right).$$

We must thus have

$$\epsilon \geq \left(\frac{1}{\delta} - 1\right) w \geq \left(\frac{1}{\delta} - 1\right) \underline{w},$$

which is false for ϵ sufficiently small. □

The idea behind Proposition 1 is that the seller will never give any rents to the buyer with the lowest net valuation. As a result, the buyer with the lowest valuation should take the outside option, causing the set of waiting buyers to unravel from the bottom.[3]

Proposition 1 shows that when the seller chooses prices over time, it attains the same profits as if they committed to a sequence of prices. Applying Stokey (1979), the seller makes the same profits if it chooses the optimal mechanism at time $t = 0$. Hence Proposition 1 implies that, with outside options, the sequentially optimal mechanism can be implemented in prices, as in Skreta (2006), and coincides with the optimal commitment mechanism. This result also suggests that the introduction of outside options has applications outside the simple Coase conjecture setting: for example, when selling one good over time to multiple agents (e.g. McAfee and Vincent (1997)), the Myerson auction should be the unique sequentially optimal mechanism.

3 Search

In Section 2 we examined a single seller facing a single buyer with outside option w. We now endogenize the outside option by considering a model of sequential search and show that there is a unique equilibrium where sellers charge the monopoly price against their residual demand.

The market consists of mass one of ex-ante identical sellers, each with zero marginal cost, and mass one of buyers. To allow for correlation between buyers' values at different sellers, suppose buyers have types $\theta \in \{\theta_1, \ldots, \theta_K\}$ where $\Pr(\theta = \theta_k) = g_k$. A type θ buyer then has value at seller i, that is conditionally iid, $v_i \sim f(\cdot|\theta)$ on $[0, 1]$. A buyer's type and value are privately known. In addition, $f(\cdot|\theta)$ has full support on $[0, 1]$.[4]

Each period proceeds as follows:

1. A buyer chooses to stays at his current seller or picks a new one at random.
2. The buyer observes his value at the seller he has chosen.
3. The seller quotes a price to the buyer as a function of the time the buyer has been with the seller.

We assume that sellers adopt *relation-specific strategies* in which the price quotes do not depend on calendar time or the history outside the relationship between the seller and the buyer. This assumption reflects the idea that a seller cannot see how many other sellers an agent has visited before arriving. Denote the price sequence charged by seller i by $\{p^i_1, p^i_{2,} \ldots\}$.

Suppose buyer θ is a customer of seller i and define his outside option $w(\theta)$ to be his expected utility after leaving seller i.

Lemma 2. *In any PBE, buyer θ has an outside option $w(\theta) > 0$.*

[3] Skrzypacz (2004) remarks that an analogue of this result obtains in a setting in which all agents have the same value v. By allowing uncertainty in values, our model extends the standard Coase conjecture setting.

[4] The full support assumption implies that a buyer can always obtain positive utility by switching. If buyers' values are perfectly correlated across sellers, a buyer cannot gain by switching sellers and there is a Coase conjecture equilibrium.

Proof. Consider seller i. There exists a $v^+ < 1$ and $p^+ > 0$ such that type θ_1 buyers with $v \geq v^+$ will buy if $p \leq p^+$ no matter what future beliefs are, and no matter what other firms are doing. To see this, let $v^* = \delta E_{v|\theta_1}[\max\{v, v^*\}]$ be the cutoff point if all firms charge zero prices and let $v^+ := \frac{1}{2}(1+v^*)$. Letting w be the buyer's continuation value if he moves on to another seller, buyer v^+ will buy from seller i in period 1 if

$$v^+ - p^+ > \max\{\delta w, \delta v^+\} \geq \max\{v^*, \delta v^+\}$$

which is satisfied if $p^+ < \frac{1}{2}\min\{(1-v^*), (1-\delta)(1+v^*)\}$. Hence any seller can guarantee itself profits $k > 0$ in any PBE. Since $f(\cdot|\theta)$ has full support, Myerson (1981) implies that any mechanism that yields strictly positive profits must yield each type strictly positive rents. That is, $w(\theta) > 0$. □

Lemma 2 allows us to apply Proposition 1, and conclude that in any PBE each seller i charges a constant price p_i. It is straightforward to then solve for the optimal prices.

Proposition 2. *Suppose the hazard rate $f(v|\theta)/[1 - F(v|\theta)]$ is increasing in v for each θ. There is an essentially unique equilibrium in which each seller charges a price p satisfying*

$$p \sum_k \left[\frac{f(v^*(\theta_k)|\theta_k)}{1 - F(v^*(\theta_k)|\theta_k)}\right] g_k - 1 = 0 \tag{1}$$

A buyer θ purchases if $v \geq v^*(\theta)$, where the cutoff satisfies

$$v^*(\theta) - p = \delta E_{v|\theta}\big[\max\{v - p, v^*(\theta) - p\}\big] \tag{2}$$

Proof. Suppose seller i chooses p_i and other sellers charge some distribution of prices inducing continuation value $w(\theta)$. We can define a cutoff $v_i^*(\theta)$ where buyer θ is indifferent between buying from seller i and continuing his search, i.e. $v_i^*(\theta) - p_i = \delta w(\theta)$. Seller i's profit per buyer is thus proportional to

$$\Pi = p_i \sum_k [1 - F(v_i^*(\theta_k)|\theta_k)]\tilde{g}_k = p_i \sum_k [1 - F(\delta w(\theta_k) + p_i|\theta_k)]\tilde{g}_k$$

where $\tilde{g}_k$ is the probability mass of type θ_k in the market. In general, the measure $\tilde{g}_k$ will differ from g_k since buyers with low types will stay in the market longer than buyers with high types. Differentiating yields the necessary first order condition

$$\sum_k [p_i f(\delta w(\theta_k) + p_i|\theta_k) - (1 - F(\delta w(\theta_k) + p_i|\theta_k))]\,\tilde{g}_k = 0.$$

Since $f(\cdot|\theta)$ has increasing hazard, this has a unique solution, implying that any equilibrium must be symmetric. In such a symmetric equilibrium the first-order condition becomes,

$$\sum_k \left[pf(v^*(\theta_k)|\theta_k) - (1 - F(v^*(\theta_k)|\theta_k))\right] \tilde{g}_k = 0. \tag{3}$$

Given we have a symmetric equilibrium, we can pin down $\tilde{g}_k$. The measure of type θ_k agents remaining after refusing to buy from t sellers is $F(v^*(\theta_k)|\theta_k)^t g_k$. Hence the probability mass of type θ_k in the market equals

$$\tilde{g}_k = \gamma \sum_{t=0}^{\infty} F(v^*(\theta_k)|\theta_k)^t g_k = \frac{\gamma g_k}{1 - F(v^*(\theta_k)|\theta_k)} \tag{4}$$

where γ is a normalizing constant. Note that this distribution depends on the price charged by other sellers via $v^*(\theta)$.

In equilibrium, the cutoff itself satisfies $\delta w(\theta) = v^* - p$, or (2). Returning to the first order condition, substituting (4) into (3) yields (1). Equations (1) and (2) thus pin down the prices and cutoffs. The equilibrium is unique: an increase in p raises the cutoff and, since $f(\cdot|\theta)$ has increasing hazard, strictly raises the right-hand side of (1). □

With competition via sequential search, the Coase conjecture fails and sellers set a constant price. In equilibrium the sellers all choose the same price and sell to measure 1 of customers. The pricing formula (1) says that when seller i lowers its price by ϵ, it loses ϵ on all its current customers (measure 1) but gains sales from the marginal customers (measure $f(v^*)$) of all those who visit it (measure $1/(1 - F(v^*))$).

An increase in δ raises the competition between firms, lowering prices p. To see this observe that, fixing p, an increase in δ, raises $v^*(\theta; p, \delta)$ point-wise. Hence p needs to increase in order for (1) to be satisfied with equality. In the limit, as $\delta \to 1$ we obtain perfect competition: $v^*(\theta; p, \delta) \to 1$ point-wise, $f(v^*)/(1 - F(v^*)) \to \infty$ and so the equilibrium price converges to 0. From the buyer's perspective, limited competition reduces their utility since the unwillingness of fellow buyers to wait helps sellers commit to high prices; however, buyers still benefit from lots of competition.

References

[1] Anderson, S. and R. Renault (1999): "Pricing, Product Diversity, and Search Costs: A Bertrand-Chamberlin-Diamond Model," *Rand Journal of Economics*, 30(4), 719–735.

[2] Ausubel, L. and R. Deneckere (1987): "One is Almost Enough for Monopoly," *Rand Journal of Economics*, 18(2), 255–274.

[3] Ausubel, L. and R. Deneckere (1989): "Reputation in Bargaining and Durable Goods Monopoly," *Econometrica*, 57(3), 511–531.

[4] Bond, E. and L. Samuelson (1984): "Durable Good Monopolies with Rational Expectations and Replacement Sales," *Rand Journal of Economics*, 15(3), 336–345.

[5] Gul, F. (1987): "Noncooperative Collusion in Durable Goods Oligopoly", *Rand Journal of Economics*, 18(2), 248–254.

[6] Gul, F., H. Sonnenschein, and R. Wilson (1986): "Foundations of Dynamic Monopoly and the Coase Conjecture," *Journal of Economic Theory*, 39, 155-190.

[7] Fuchs, W. and A. Skrzypacz (2010): "Bargaining with Arrival of New Traders", *American Economic Review,* 100(3), 802–36.

[8] Fudenberg, D., D. Levine and J. Tirole (1987): "Incomplete Information Bargaining with Outside Opportunities" *Quarterly Journal of Economics*, 102(1), 37–50.

[9] Kahn, C. (1986): "The Durable Goods Monopolist and Consistency with Increasing Costs", *Econometrica*, 54(2), 275–294.

[10] McAfee, R. P. and D. Vincent (1997): "Sequentially Optimal Auctions", *Games and Economic Behavior*, 18(2), 246–276.

[11] Myerson, R. (1981): "Optimal Auction Design", *Mathematics of Operations Research*, 6(1), 58–73.

[12] Sobel, J. (1991): "Durable Goods Monopoly with Entry of New Consumers Durable Goods Monopoly with Entry of New Consumers", *Econometrica*, 59(5), 1455–1485.

[13] Skreta, V. (2006): "Sequentially Optimal Mechanisms", *Review of Economic Studies*, 73(4), 1085–1111.

[14] Skrzypacz, A. (2004): "Bargaining under Asymmetric Information and the Hold-Up Problem", unpublished manuscript, Stanford GSB.

[15] Stokey, N. (1979): "Intertemporal Price Discrimination", *Quarterly Journal of Economics*, 93(3), 355–371.